Exercise 50

Let $P(t)$ be the percentage of Americans under the age of 18 at time t. The table gives values of this function in census years from 1950 to 2010.

t	$P(t)$	t	$P(t)$
1950	31.1	1990	25.7
1960	35.7	2000	25.7
1970	34.0	2010	24.0
1980	28.0		

(a) What is the meaning of $P'(t)$? What are its units?

(b) Construct a table of estimated values for $P'(t)$.

(c) Graph P and P'.

(d) How would it be possible to get more accurate values for P'?

Solution

Part (a)

$P'(t)$ represents the rate that the percentage of Americans increases as t increases. Its units are percentage points per year.

Part (b)

Start by calculating the slopes of the secant lines.

$$m_5 = \frac{P(1960) - P(1950)}{1960 - 1950} = \frac{35.7 - 31.1}{10} = 0.46$$

$$m_6 = \frac{P(1970) - P(1960)}{1970 - 1960} = \frac{34.0 - 35.7}{10} = -0.17$$

$$m_7 = \frac{P(1980) - P(1970)}{1980 - 1970} = \frac{28.0 - 34.0}{10} = -0.60$$

$$m_8 = \frac{P(1990) - P(1980)}{1990 - 1980} = \frac{25.7 - 28.0}{10} = -0.23$$

$$m_9 = \frac{P(2000) - P(1990)}{2000 - 1990} = \frac{25.7 - 25.7}{10} = 0.00$$

$$m_0 = \frac{P(2010) - P(2000)}{2010 - 2000} = \frac{24.0 - 25.7}{10} = -0.17$$

For the years, 1960–2000, take the average of the secant lines to get the best estimate for $P'(t)$.

$$P'(1950) \approx m_5 = 0.46$$

$$P'(1960) \approx \frac{m_5 + m_6}{2} = 0.145$$

$$P'(1970) \approx \frac{m_6 + m_7}{2} = -0.385$$

$$P'(1980) \approx \frac{m_7 + m_8}{2} = -0.415$$

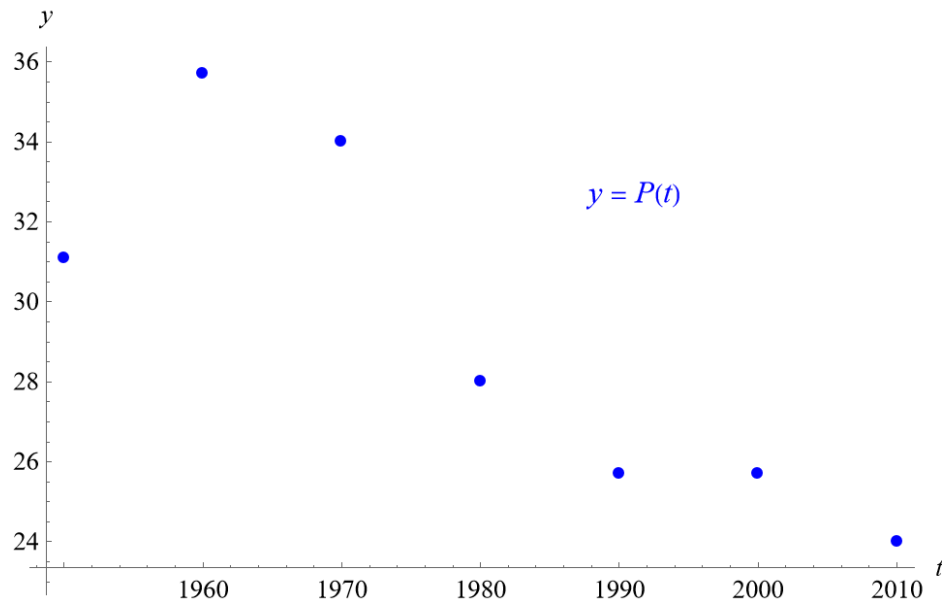
$$P'(1990) \approx \frac{m_8 + m_9}{2} = -0.115$$

$$P'(2000) \approx \frac{m_9 + m_0}{2} = -0.085$$

$$P'(2010) \approx m_0 = -0.17$$

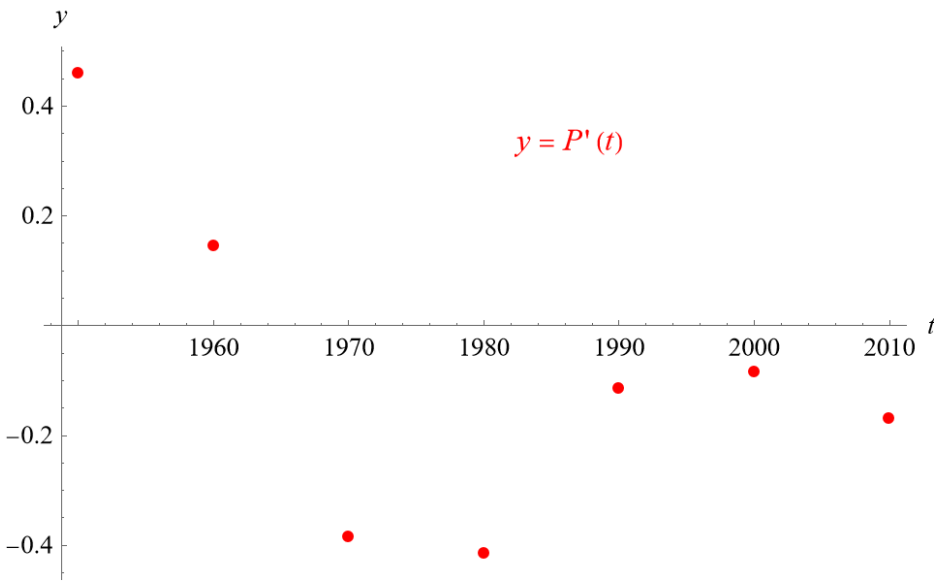
Part (c)

Below is a graph of $P(t)$ versus t.



This graph shows the percentage of the American population under the age of 18.

Below is a graph of $P'(t)$ versus t.



This graph shows the change in the percentage of the American population under the age of 18.

Part (d)

To get more accurate results for $P'(t)$, the population would have to be known more often than every 10 years.